

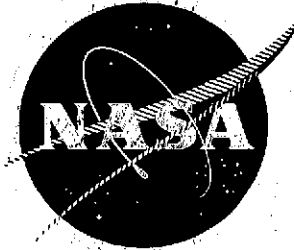
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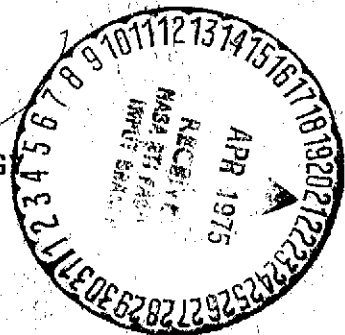
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SUMMARY

In this report the results of an experimental investigation undertaken to determine the frequency dependence of the response factors of various gaseous propellant rocket injectors subject to axial instabilities are presented. The injector response factors were determined, using the modified impedance-tube technique, under cold-flow conditions simulating those observed in unstable rocket motors. The tested injectors included a gaseous-fuel injector element, a gaseous-oxidizer injector element and a coaxial injector with both fuel and oxidizer elements. Emphasis was given to the determination of the dependence of the injector response factor upon the open-area ratio of the injector, the length of the injector orifice, and the pressure drop across the injector orifices. The measured data are shown to be in reasonable agreement with the corresponding injector response factor data predicted by the Feiler and Heidmann model.

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INTRODUCTION

The stability of the combustor of a rocket motor depends upon the wave-energy balance between the various gain and loss mechanisms that are present in the system. The primary source of wave-energy gain is the combustion process. Wave-energy losses are provided by the mean flow, the nozzle, and mechanical damping devices (e.g., acoustic liners) which may be present in the system. As the stability of a rocket motor depends upon the difference between the gain and loss mechanisms, it is of utmost importance that quantitative data capable of describing the damping provided by the loss mechanisms and the driving provided by the unsteady combustion process must be available. Furthermore, an understanding of the dependence of these gain and loss mechanisms upon engine design parameters and operating conditions is needed. The investigation described in this report was undertaken for the purpose of obtaining a better understanding of the driving provided by the unsteady combustion process; specifically, this investigation was concerned with the acquisition of experimental data that quantitatively describes the manner in which various injector designs affect the energy gain provided by the unsteady combustion process.

The injector elements of a gaseous rocket motor control the steady state gas flow and heat transfer patterns inside the combustion chamber. In addition, the injector design influences the response of the flow rate through the injector to combustion chamber disturbances. The characteristics of this response have a profound effect upon engine stability. Customarily, the influence of the injector upon the chamber stability is described by an injector response factor which describes the manner in which the propellants' burning rate responds to a given pressure oscillation in the chamber. The injector response factor basically accounts for the dependence of the unsteady burning rate upon both the unsteady combustion process and unsteady flow of propellants through the injector elements. This response factor can be used to evaluate the energy added by the combustion process into the disturbance in the combustion chamber. It can also be used as the injector

end boundary condition that needs to be satisfied in a stability analysis of a gaseous rocket combustion chamber.

Most of the available experimental investigations¹⁻⁷ on the behavior of gaseous propellant injectors were concerned with the steady operation of these devices with little or no consideration being given to the corresponding unsteady problem. In contrast, the analytical studies of Feiler and Heidmann were concerned with the predictions of the characteristics of the response factor of a gaseous injector element. In the Feiler and Heidmann analysis,^{8,9} a single gaseous hydrogen injector element is modeled as a combination of lumped flow elements. The desired expressions for the injector response factor are then obtained by solving the conservation equations that describe the unsteady flow inside the various components of the injector. The resulting expressions describe the dependence of the injector response factor upon the injector geometry and the flow conditions in the chamber and the injector. In this analytical model, combustion is assumed to be concentrated in front of the injector face and the effects of mixing and chemical reactions are accounted for by the introduction of an as yet unknown time delay τ_b^* . The period τ_b^* describes the time required for the gaseous oxidizer and fuel streams to mix and burn. In Ref. 10, the Feiler and Heidmann predictions⁸ have been modified to account for the compressibility of the gaseous streams flowing through the injector elements.

The results of Refs. 8 and 10 indicate that for a given frequency range and for certain ranges of the parameter τ_b^* , various injector designs can indeed result in the amplification of chamber disturbances. When τ_b^* is identically zero, which corresponds to the case of no combustion present in the system, the results of Refs. 8 and 10 indicate that under these conditions the injector acts as a mechanical damping device; a situation that is to be expected from related studies of Helmholtz resonators and acoustic liners.

Although the predictions of the Feiler and Heidmann analysis have been known for a number of years, they have never been verified experimentally. It is one of the objectives of this investigation to provide

experimental data that could be used to check the validity of the Feiler and Heidmann model. In addition, this investigation is concerned with providing experimental data that will quantitatively describe the manner in which various coaxial injector designs affect the stability of gaseous propellant rocket motors. In pursuit of the above-mentioned objectives, the response factors of a number of gaseous rocket injector configurations have been measured under cold-flow conditions simulating those observed in rocket motors experiencing axial instabilities. Specifically, the response factor of configurations that simulate the flow conditions in a gaseous-fuel injector element, a gaseous-oxidizer injector element, and a coaxial injector with both fuel and oxidizer elements have been determined using the modified impedance-tube technique. The measured injector response factor data are presented and the results discussed in this report.

NOMENCLATURE

A	area
C	Capacitance, defined by Eq. (4)
c	speed of sound
I	Inductance, defined by Eq. (4)
L	length of the injector orifice
l_{eff}	effective orifice length given by Eq. (14)
M	Mach number
N	nondimensional injector response factor
P	pressure
R	Resistance, defined by Eq. (4)
V	injector dome volume
W	mass flow rate of propellant
Y	admittance
y	nondimensional admittance
α	admittance parameter defined by Eq. (7)
β	admittance parameter defined by Eq. (8)

γ	specific heat ratio
δ	equal to $(\bar{P}_d^* - \bar{P}_c^*)/\bar{P}_c^*$
λ	wavelength
ρ	density
σ	open-area ratio of the injector
τ	time lag
ω	angular frequency

Superscripts

$(\bar{})$	steady state quantity
$()^*$	dimensional quantity
$()'$	perturbation quantity

Subscripts

$()_b$	associated with the combustion process
$()_c$	evaluated in the chamber
$()_d$	evaluated in the injector dome
$()_f$	associated with the fuel
$()_{ox}$	associated with the oxidizer
$()_s$	evaluated at the injector surface
$()_1$	evaluated at injector orifice entrance
$()_2$	evaluated at injector orifice exit

ANALYTICAL CONSIDERATIONS

The ability to quantitatively describe the injector response factor is of great practical importance since the combined response of the injector flow rate and the combustion process to chamber disturbances is the mechanism responsible for amplifying and maintaining combustion instability oscillations. In an effort to develop an analytical technique for the prediction of the response factor of a gaseous injector,

Feiler and Heidmann^{8, 9} analyzed in detail the unsteady flow through the gaseous hydrogen injector element shown in Fig. 1. Combustion is assumed to occur a certain distance downstream of the injector exit plane and the response of the injector flow rate to a small amplitude pressure oscillation in the chamber is determined by analyzing the linearized conservation equations for each of the injector components. Assuming that each of the injector components behaves as a lumped element, and applying the Laplace transform to the linearized conservation equations, the relationships presented in Fig. 1 are obtained. By appropriate manipulations of these equations and setting the Laplace operator s equal to $i\omega$, which implies a sinusoidal time dependence of the perturbations, the following expression for the injector response factor was obtained:

$$N = \frac{W'_b}{P'_c} = \left(\frac{W'_{b \max}}{P'_{c \max}} \right) e^{i\theta} \quad (1)$$

where

$$\frac{W'_{b \max}}{P'_{c \max}} = \frac{-1}{R_2 \left\{ \left[\frac{R_1}{C^* \omega^*} - I^* \omega^* \right]^2 + \left[2 \left(\frac{R_1 \Delta P_1^*}{\bar{P}_d^*} + \frac{\Delta P_2^*}{\bar{P}_2^*} \right) \right]^2 \right\}^{\frac{1}{2}}} \quad (2)$$

$$\theta = \frac{\pi}{2} - \omega^* \tau_b^* - \arctan \frac{2 \left\{ \frac{R_1 \Delta P_1^*}{\bar{P}_d^*} + \frac{\Delta P_2^*}{\bar{P}_2^*} \right\}}{\left\{ \frac{R_1}{C^* \omega^*} - I^* \omega^* \right\}} \quad (3)$$

and

$$C^* \omega^* = \left(\bar{p}_d^* V^* / \gamma \bar{W}^* \right) \omega^* ; \quad I^* \omega^* = \left[\bar{W}^* (L^* / A_1^*) / g \bar{P}_2^* \right] \omega^* \quad (4a)$$

$$\frac{\Delta P_1^*}{\bar{P}_d^*} = (\bar{P}_d^* - \bar{P}_1^*)/\bar{P}_d^* ; \quad \frac{\Delta P_2^*}{\bar{P}_2^*} = (\bar{P}_2^* - \bar{P}_c^*)/\bar{P}_2^* \quad (4b)$$

$$R_1 = \frac{\bar{P}_d^*}{\bar{P}_1^* - (\Delta P_1^*/\gamma)} ; \quad R_2 = \frac{\bar{P}_2^*}{\bar{P}_c^* - (\Delta P_2^*/\gamma)} \quad (4c)$$

The quantity τ_b^* appearing in Eq. (3) is the residence time of a propellant mass element in the combustor prior to its combustion; τ_b^* is identically zero when there is no combustion in the system. The parameters appearing in Eq. (4) depend upon the injector geometry and engine operating conditions, and their influence upon the injector element response factor is also of interest to rocket designers.

Expressions similar to those developed above for the gaseous-fuel injector element can also be developed for the gaseous-oxidizer injector element. The total response, N_t , of a coaxial gaseous injector element can then be obtained, by substituting the expressions for the fuel and oxidizer response factors into the following equation:

$$N_t = \frac{W_t'}{P'} = \frac{\left(\frac{W_t^*}{\bar{W}_t^*}\right)'}{\left(\frac{P^*}{\bar{P}^*}\right)'} = \frac{\left\{\left(\frac{W_{ox}^*}{\bar{W}_{ox}^*}\right)' + \left(\frac{W_f^*}{\bar{W}_f^*}\right)'\right\}}{\left(\frac{P^*}{\bar{P}^*}\right)'} \quad (5)$$

$$= \left[\frac{\bar{W}_{ox}^*}{\bar{W}_t^*} \right] N_{ox} + \left[\frac{\bar{W}_f^*}{\bar{W}_t^*} \right] N_f \quad (5)$$

where N_{ox} and N_f respectively represent the response factors of the oxidizer and fuel injector elements while $\bar{W}_{ox}^*/\bar{W}_t^*$ and \bar{W}_f^*/\bar{W}_t^* represent the ratios of the mean oxidizer and fuel flow and the total mean flow, respectively.

RESPONSE FACTOR DETERMINATION

The required injector response factor data were determined in this investigation from injector admittance data measured by use of the modified impedance-tube technique. The impedance tube setup shown in Fig. 2, consists of a 6-inch diameter cylindrical tube with a sound source capable of generating harmonic waves of desired frequency placed at one end. The injector element under investigation is placed at the other end. During an experiment, the flow of a gaseous propellant through the injector is simulated by the flow of air. Regulating valves are provided to ensure that the pressure drop across the injector orifices is maintained at a required value. By means of an acoustic driver, a standing wave pattern of a given frequency is excited in the tube and a microphone probe is traversed along the tube to measure the axial variation of the standing pressure wave pattern. As explained in the next section, the admittance of the injector end of the impedance-tube is determined from the measured axial variation of the standing pressure wave. The frequency dependence of the admittance and the response factor of the injector is determined by repeating the experiment at different frequencies.

The first step in the determination of the injector response factor N consists of the measurement of the "average" surface admittance Y_s^* at the injector end of the modified impedance tube. The "average" surface admittance is defined as the ratio of the "average" normal velocity perturbation across the injector surface and the local pressure perturbation; that is:

$$Y_s^* = \frac{\bar{u}_s^{*'} \cdot \bar{n}}{P_s^{*'}} \quad (6)$$

The admittance Y_s^* is a complex number whose real and imaginary parts describe the relationships that exist at the location under consideration between the amplitudes and phases of the velocity and pressure perturbations.

From a physical point of view it is more satisfying to describe the admittance by means of two parameters α and β which respectively describe changes in amplitudes and phases between the incident and reflected pressure waves at the location under consideration; that is:

$$\left[\frac{\text{Amplitude of Reflected Pressure Wave}}{\text{Amplitude of Incident Pressure Wave}} \right]_{\text{Injector Face}} = e^{-2\pi\alpha} \quad (7)$$

$$\left[\frac{\text{Phase change Between Incident and}}{\text{Reflected Pressure Waves}} \right]_{\text{Injector Face}} = \pi(1 + 2\beta) \quad (8)$$

The parameter β appearing above satisfies the condition $|\beta| \leq 0.5$.

The expressions required for the calculation of the injector surface admittance are obtained from solutions of the system of conservation equations which describe the behavior of small amplitude, one-dimensional waves inside an impedance-tube containing a steady one-dimensional flow. These solutions are required to satisfy an admittance boundary condition at the injector surface in terms of the as yet unknown parameters α and β . The resulting expressions (See Ref. 12 for detailed derivations of these solutions), describing the time and space dependence of the pressure and velocity perturbations at the injector surface, are substituted into Eq. (6) to obtain an expression for the injector surface admittance. Normalizing the resulting expression with the characteristic admittance $Y_g^* = 1/\rho^* c^*$ of the gas medium, the following expression for the nondimensional injector surface admittance y_s is obtained¹²:

$$y_s = \frac{Y_s^*}{Y_g^*} = \Gamma + i\eta = \coth \pi(\alpha - i\beta) \quad (9)$$

It can also be shown¹² that the parameters α and β , which appear in Eqs. (7), (8) and (9) must satisfy the following relationships be-

tween variables describing the characteristics of the standing wave pattern:

$$\alpha = \frac{1}{\pi} \tanh^{-1} \left[\frac{|P_{\min}^*|}{|P_{\max}^*|} \right]; \quad \beta = \frac{2Z_{\min}^*}{\lambda^*} \quad (10)$$

In impedance-tube experiments and in the present study, the relationships presented in Eq. (10) are used to determine the admittance variables α and β . The procedure leading to the determination of α and β consists of measuring (a) the distance Z_{\min}^* from the injector surface to the first pressure amplitude minimum and (b) the ratio of $|P_{\min}^*|/|P_{\max}^*|$ of the minimum pressure amplitude to the maximum pressure amplitude. The resulting values of α and β are then substituted into Eq. (9) to obtain the injector surface admittance.

From the measured injector surface admittance y_s , the injector orifice admittance y_2 is determined by using the following relationship obtained from the perturbed form of mass conservation law:

$$(u^*)_s' A_s^* = (u^*)_2' A_2^*$$

which upon dividing by $(P^*)_s'$ gives

$$y_2 = y_s / \sigma \quad (11)$$

where $\sigma = A_2^*/A_s^*$ is the injector open-area ratio. In deriving Eq. (11) the gas has been assumed to be incompressible; an allowable assumption for the situation under consideration.

An expression relating the nondimensional response factor N to the nondimensional admittance y is obtained from the definitions of these two quantities as follows:

$$\begin{aligned}
N &= \frac{\vec{W}^{*'} \cdot \vec{n} / \bar{W}^*}{P^{*'} / \bar{P}^*} = \frac{\bar{P}^*}{\bar{\rho}^{*'} \bar{u}^*} \left[\bar{\rho}^{*'} \frac{\vec{u}^{*'} \cdot \vec{n}}{P^{*'}} + \frac{\bar{\rho}^{*'}}{P^{*'}} \vec{u}^* \cdot \vec{n} \right] \\
&= \frac{1}{\gamma \bar{M}} \left[\bar{\rho}^{*'} \frac{\vec{u}^{*'} \cdot \vec{n}}{P^{*'}} + \bar{M} \cdot \vec{n} \right] \\
&= \frac{1}{\gamma \bar{M}} (\gamma + \bar{M} \cdot \vec{n}) \tag{12}
\end{aligned}$$

In deriving Eq. (12) it has been assumed that the gas is perfect and that the oscillations are isentropic. The response factor N of the test injectors is finally obtained by substituting the measured orifice admittance y_2 into Eq. (12) which can be rewritten in the following form for the experimental setup of this investigation:

$$N = \frac{1}{\gamma} \left[- \frac{y_2}{\bar{M}_2} + 1 \right] \tag{13}$$

TEST INJECTORS

In order to obtain the needed data, the frequency dependence of the response factors of the injector configurations shown in Figs. 3 through 6 have been determined. The characteristic dimensions of these injectors, namely, the injector orifice open-area ratio, the orifice length, and the injector dome volume are also presented in the above-mentioned figures.

Injector configurations 1 and 2 were designed to simulate the flow behavior through gaseous-fuel injector elements. The dimensions of these configurations were chosen to provide data capable of determining the effect of the injector open-area ratio upon the injector response factor. Injector configurations 3 through 5 were designed to simulate the flow behavior in gaseous-oxidizer injector elements, and their

dimensions were chosen to allow the determination of the dependence of the injector response factor upon the orifice length. Injector configuration 6, shown in Fig. 6, consists of a combination of configurations 1 and 3. This configuration was designed to simulate the flow behavior in a coaxial injector of a gaseous rocket motor. This injector configuration was tested to check the validity of Eq. (5) by comparing its measured response factors with predicted response factor data obtained by substituting the individually-predicted response factors of configurations 1 and 3 into Eq. (5).

RESULTS

Introduction

The results presented in this section were obtained by measuring the admittances and response factors of the test injectors over the frequency range of 150 to 800 Hz which included their resonant frequency. To establish the repeatability of the experimental data, the frequency dependence of the response factor of one of the test injectors was measured on two different occasions and the response factor data obtained in these tests are presented in Fig. 7. An examination of this figure indicates that the measurement technique yields repeatable data. The scatter observed in the measured values of the imaginary part of the response factor is due to the fact that at the corresponding frequencies the standing wave in the impedance tube had a flat minima and hence its axial location could not be precisely measured.

Before presenting the results, it is necessary to point out a difference between the geometrical configurations of the injector elements whose admittances were measured in this study and the injector configurations considered in the theoretical model of Feiler and Heidmann.⁸ The theoretical analysis considers the behavior of a single injector element and its predictions provide a response factor that is valid at the exit plane of the injector orifice. As it would be extremely difficult to directly measure the response factor of a single injector element, this study undertook the measurement of the response

factors of configurations containing either 5 or 13 injector elements. As stated earlier, the admittances measured in this study represent "average" admittances over the tested injector surface. Hence, before any meaningful comparisons between the predicted and the measured sets of admittance data can be made, the above-mentioned difference must be suitably taken into consideration. This point was discussed in the previous section where it was shown that by using mass conservation considerations, this difference can be accounted for by multiplying the theoretically predicted orifice admittances by the open-area ratio σ of the injector configuration. This step "averages" the predicted orifice admittance over the injector surface. To illustrate this point, the theoretically predicted frequency dependence of the admittances of injector configuration 1 with a pressure drop δ of 0.068 across the injector orifices is presented in Fig. 8. The broken lines in this figure describe the admittances at the exit plane of the injector orifices while the solid lines represent the "average" admittances of the injector surface. It is this "average" data which has to be compared with the admittances measured during this investigation.

In the present study, the expressions provided by Feiler and Heidmann⁸ have been slightly modified when used to compute the predicted admittances and response factors of the test injector configurations. This was necessitated by the observation that the measured resonant frequencies of the tested injectors did not coincide with their predicted values. This is illustrated by the data presented in Fig. 9. The broken line in this figure describes the theoretically predicted frequency dependence of the real and imaginary parts of the response factor of one of the test injectors. An examination of this figure indicates that while the two sets of data are similar in magnitude and shape, the observed injector resonant frequency is lower than its predicted value. In an effort to explain this frequency shift, use was made of knowledge developed in studies concerned with the behavior of Helmholtz resonators and acoustic liners^{13, 14} where it has been well known that the effective length of the slug of the gaseous mass oscillating within the orifice is longer than the orifice length.

It is also well known that the resonant frequencies of Helmholtz resonators and acoustic liners are inversely proportional to the square root of the orifice length. This suggests that the actual length L^* of the injector orifices should be replaced by an effective length l_{eff}^* whenever it appears in the analytical expressions of the Feiler and Heidmann analysis. From experimental reactance data of acoustic liners with apertures of various thicknesses, Garrison¹³ developed the following empirical relation for the effective length l_{eff}^* :

$$l_{eff}^* = L^* + 0.85 \left[1 - 0.70\sqrt{\sigma} \right] (D_o^* - D_i^*) \quad (14)$$

where D_o^* and D_i^* are respectively the outer and inner diameters of the orifices. Computing the predicted response factor data of the test injector with L^* replaced by the effective length l_{eff}^* , the result indicated by the solid line in Fig. 9 was obtained. The experimental resonant frequency now is in better agreement with the predicted resonant frequency than the original Feiler and Heidmann prediction. Based on this result all of the theoretically predicted data presented in the remainder of this report was obtained by suitably incorporating Eq. (14) into the expressions of Ref. 8.

Comparison of Measured and Predicted Injector Admittances

The injector admittances measured during the course of the present study are presented in Figs. 10 through 14 along with admittance data predicted by the Feiler and Heidmann model. These figures describe, respectively, the frequency dependence of the real and imaginary parts of the surface admittances of injector configurations 1 through 5. An examination of these figures indicates a reasonable agreement between the measured and predicted admittances. The discrepancy observed in the data may be, among other factors, due to the fact that radial pressure gradients were measured in the domes of some of the tested injectors. These pressure gradients resulted in different pressure drops across different injector elements. The possibility of such pressure

gradients is not considered in the theoretical model⁸ and their effect cannot be accounted for in predicting the injectors' response factors. The theoretical admittances obtained in this study were computed assuming that the pressure drops across all of the injector orifices were equal to the pressure drop measured across one of the outer injector elements; an assumption that is contrary to the above-mentioned observations.

The response factors of injector configurations 1 through 5 were obtained by substituting the measured admittance data into Eq. (13). As suggested in Ref. 8, the response factor data for the injectors tested in this program, with different pressure drops, are plotted in Fig. 15 in terms of a generalized response factor ϕ defined as

$$\phi = N_{\text{Real}} \left\{ 2R_2 \left(\frac{R_1 \Delta P_1^*}{\bar{P}_d^*} + \frac{\Delta P_2^*}{\bar{P}_2^*} \right) \right\} \quad (15)$$

and a generalized reactance Ψ defined as

$$\Psi = \left(\frac{R_1}{C^* \omega^*} - I^* \omega^* \right) / 2 \left(\frac{R_1 \Delta P_1^*}{\bar{P}_2^*} + \frac{\Delta P_2^*}{\bar{P}_2^*} \right) \quad (16)$$

An examination of Fig. 15 indicates a reasonable agreement between the experimental data and the predictions of the Feiler and Heidmann model. Furthermore, this plot points to a convenient way for correlating and plotting injector response factor data.

Effect of Injector Design Parameters Upon Injector Response Factors

During this investigation, the dependence of the injector response factors upon the pressure drop across the injector orifices, the open-area ratio of the injector and the length of the injector orifices were investigated. The dependence of the injector response upon the pressure drop across the injector orifices is demonstrated by the data presented earlier in Figs. 10 through 14. An examination of these figures

indicates that the injector admittances and response factors decrease rapidly in magnitude with increase in pressure drop across the orifices. Increase in pressure drop results in an increase in the resistance of the injector plate. This decreases the coupling between the pressure oscillation inside the injector dome and the pressure oscillation in the combustor in front of the injector plate. The increase in the injector pressure drop is observed, however, to have little effect upon the resonant frequency of the injector.

In order to determine the dependence of the injector response factor upon the injector characteristic dimensions, the admittance data measured with test configurations 1, 4 and 5 were substituted into Eq. (13) and the response factors obtained are presented in Figs. 16 and 17. The data presented in Fig. 16 describes the effect of the open-area ratio upon the injector response factor for a given orifice length and mass flux through the injector orifices. An examination of Fig. 16 indicates that an increase in the open-area ratio of the injector results in an increase in the damping provided by the injector. In addition, the data indicates an increase in the resonant frequency which is to be expected from results of studies on Helmholtz resonators. The increase in the injector damping is due to the fact that for a given mass flux an increase in the open-area ratio results in a decrease in the pressure drop across the orifices. This in turn decreases the injector resistance. From a stability point of view this seems to suggest that, for a given mass flow across the injector plate, an injector should be designed with as large an open-area ratio as possible. However, in contemplating such changes in actual systems, one should also consider how an increase in the open-area ratio would affect other gain or loss mechanism in the system. For example, in an actual gaseous propellant rocket motor a decrease in the pressure drop across the injector orifices also affects the mixing rate and hence the propellants burning rate.

For a given open-area ratio and pressure drop across the orifices, data describing the effect of the orifice length upon the injector response factor is presented in Fig. 17. An examination of this figure

indicates that an increase in the orifice length from 0.875" to 1.75" resulted in a decrease in the resonant frequency of the injector. Further examination of Fig. 17 indicates that although there is no observable change in the magnitude of the response factor at resonance, an increase in the orifice length decreases the band width of the response curve.

CONCLUSIONS

The measured data indicates that under the test conditions encountered in this study, there is reasonable agreement between the measured injector response factors and those predicted by the Feiler and Heidmann model. The good agreement observed between the measured and predicted total response factors of coaxial injectors containing both fuel and oxidizer elements suggests that the procedure suggested by Feiler and Heidmann for calculating the total response factors from individual injector response factor data is indeed valid.

The measured response factor data indicates that the orifice length can be varied to shift the resonant frequency of the injector without any change in the magnitude of the response factor at resonance. However, changes in pressure drop across the orifices and the open-area ratio of the injector were found to have a considerable effect on the injector response factor.

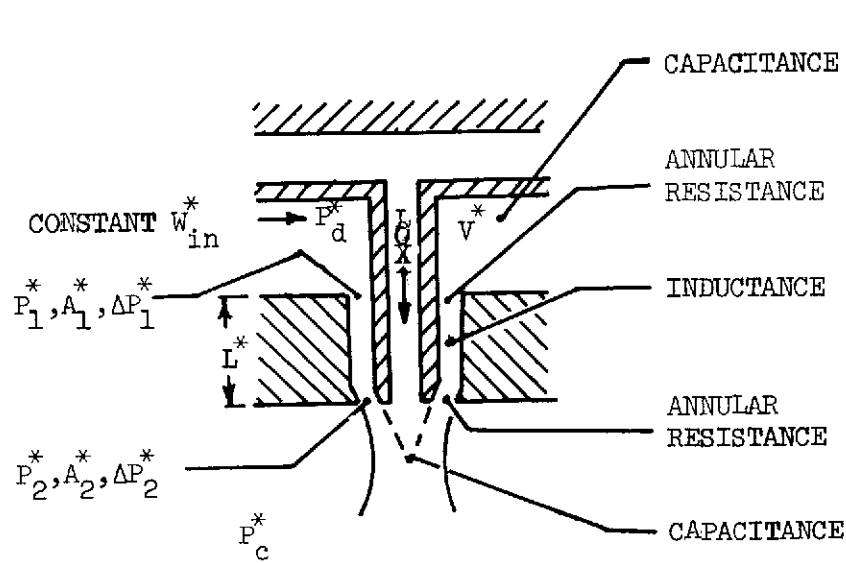
The injector configurations investigated in this program were similar to Helmholtz Resonators with a steady through flow. The interaction of such a configuration with a sound wave is not expected to produce any wave amplification, as was recognized by Feiler and Heidmann and confirmed by the data reported in this report. When a time delay, τ_p^* , due to combustion is added to the theoretical model, the phase relationship between the pressure and velocity perturbations required for wave amplification (and instability) is obtained. To test the latter hypothesis, and in the process measure the characteristic combustion time, τ_p^* , additional studies that will measure the response factors of "reacting" gaseous rocket injectors, under a variety of conditions simulating those observed in unstable engines, are needed.

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$$\frac{\bar{p}_d^* V^*}{\gamma \bar{W}^*} s P_d' = -W'$$

$$W' = \frac{1}{2} \left[\frac{\bar{P}_d^*}{\Delta P_1^*} P_d' + \left(\frac{1}{\gamma} - \frac{\bar{P}_1^*}{\Delta P_1^*} \right) P_1' \right]$$

$$P_1' - P_2' = \frac{\bar{W}^* L^*}{g \bar{P}_2^* A_1^*} s W'$$

$$W' = \frac{1}{2} \left[\frac{\bar{P}_2^*}{\Delta P_2^*} P_2' + \left(\frac{1}{\gamma} - \frac{\bar{P}_c^*}{\Delta P_2^*} \right) P_c' \right]$$

$$W_b' = W' e^{-\tau_b^* s}$$

Figure 1. Gaseous Hydrogen Injector.

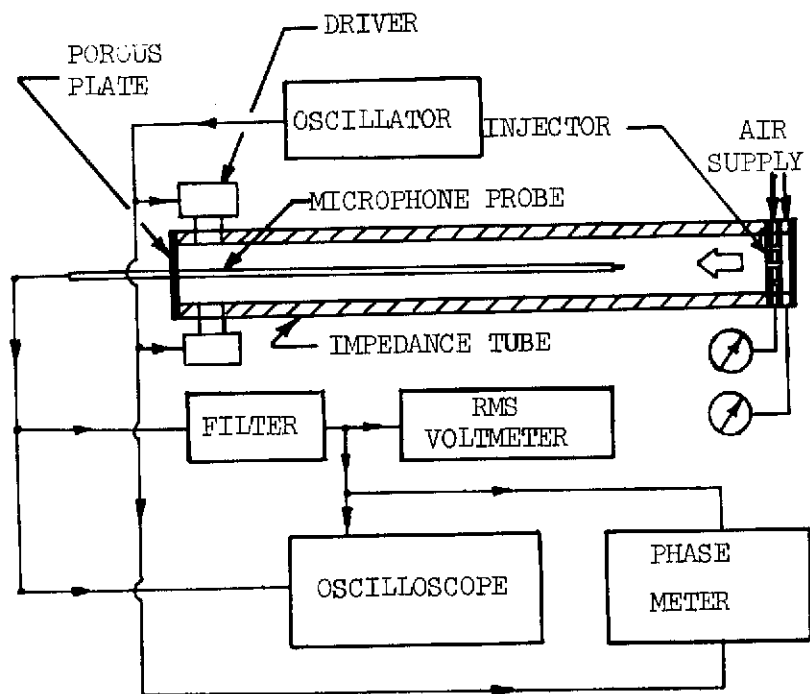
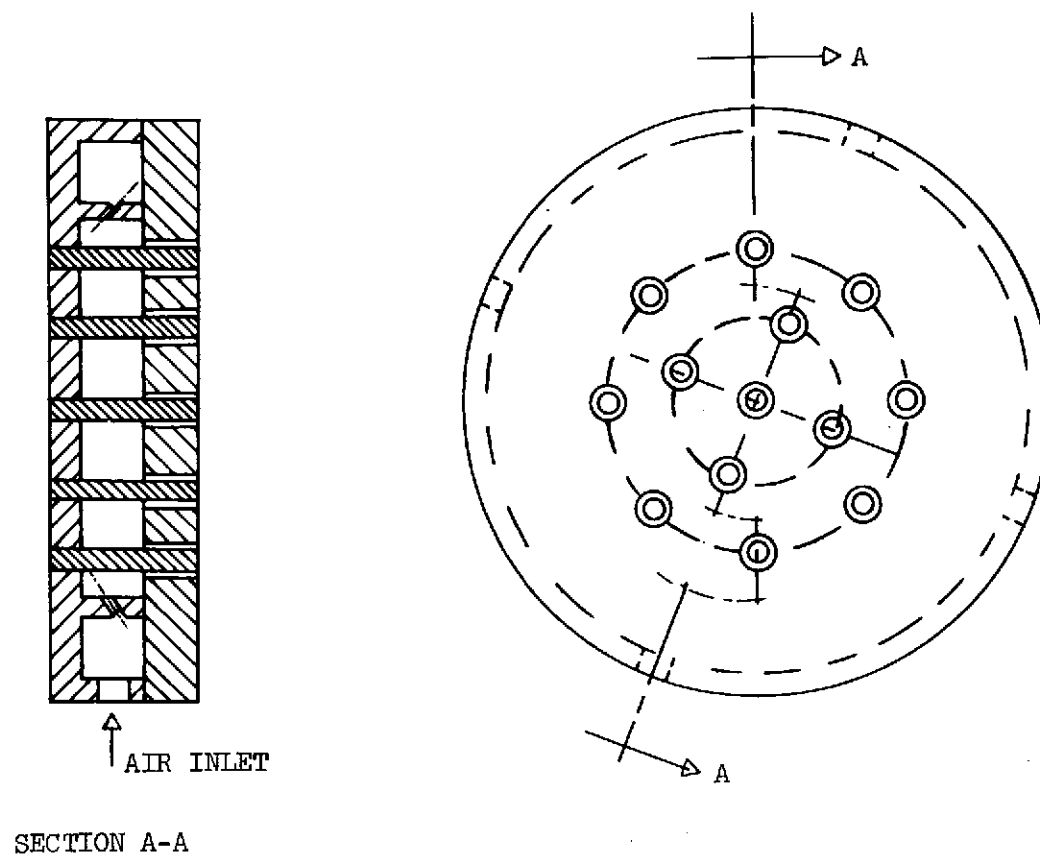


Figure 2. Experimental Apparatus

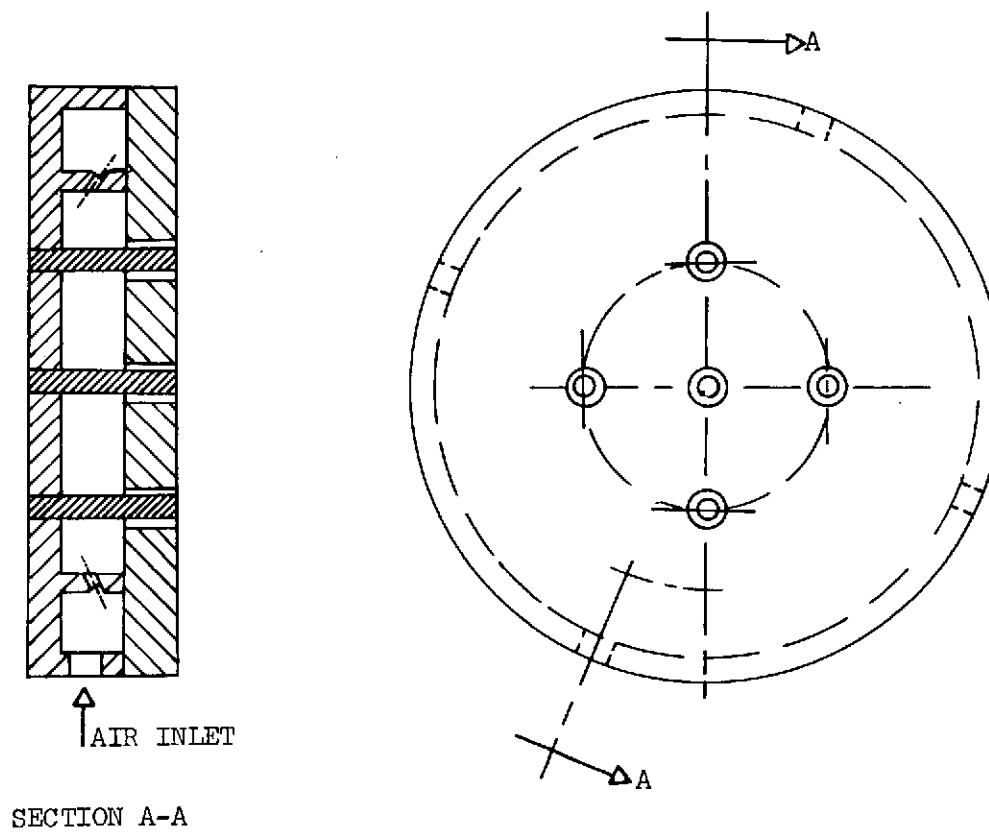
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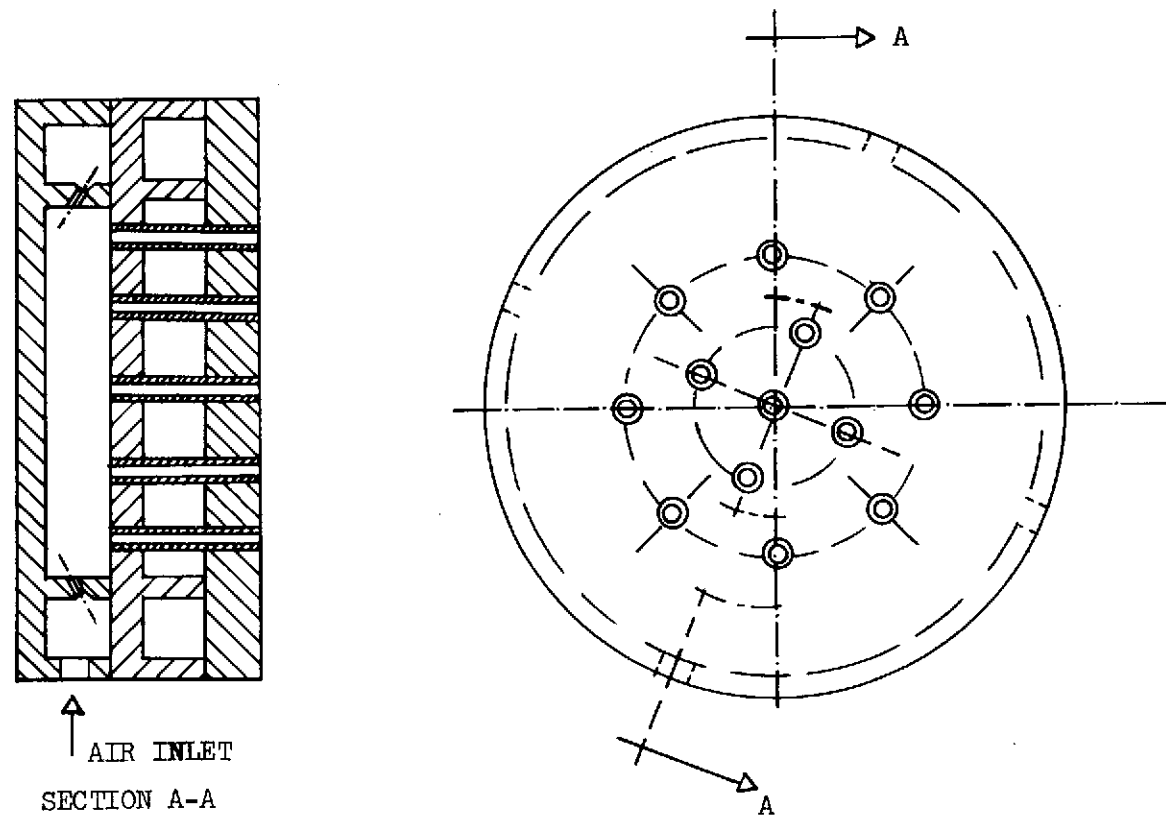
CONFIGURATION	$\sigma(\%)$	L (IN.)	V (IN. ³)
1	4.7	0.875	27.6

Figure 3. Description of Injector Configuration 1.



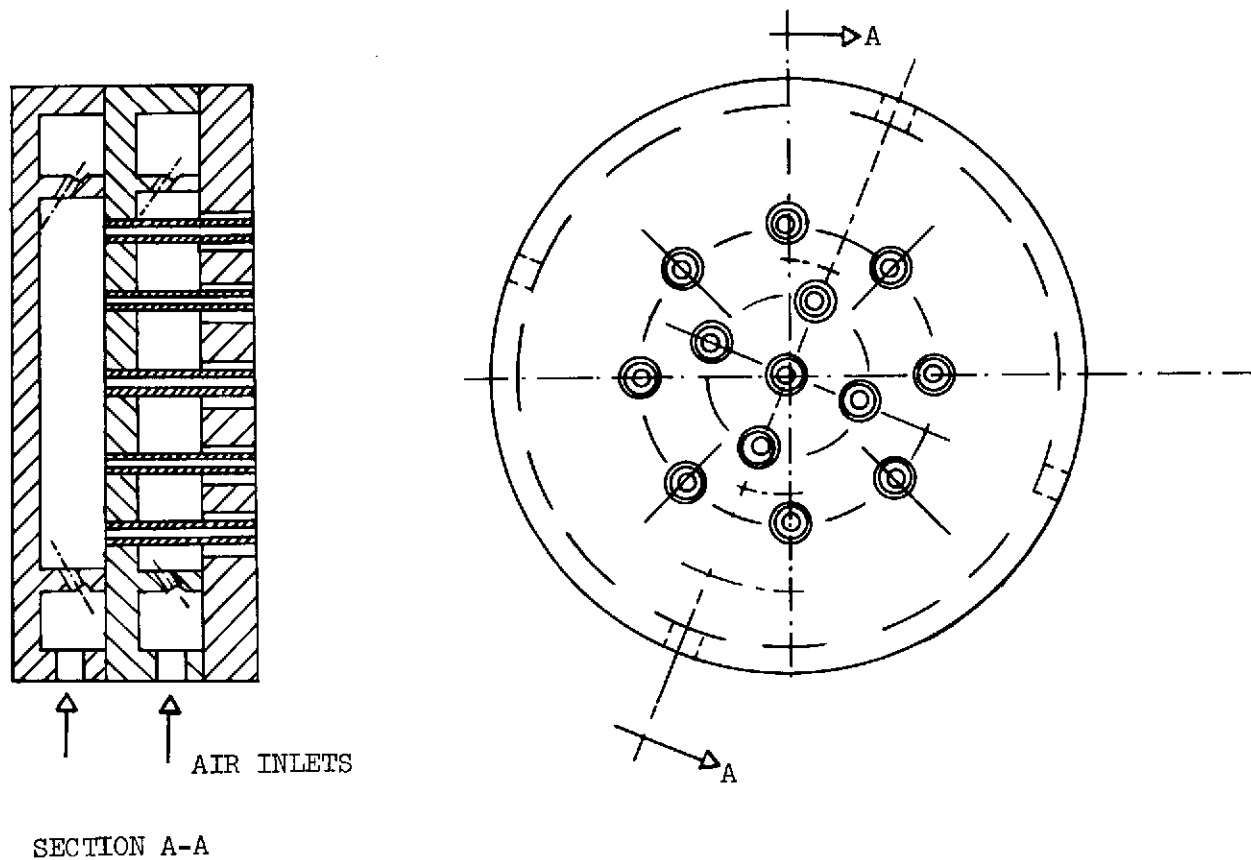
CONFIGURATION	σ (%)	L (IN.)	V (IN ³)
2	1.8	0.875	28.0

Figure 4. Description of Injector Configuration 2.



CONFIGURATION	σ (%)	L (IN.)	V (IN. ³)
3	1.7	2.38	28.2
4	10.2	0.875	28.2
5	10.2	1.75	28.2

Figure 5. Descriptions of Injector Configurations 3, 4 and 5.



CONFIGURATION		σ (%)	L (IN.)	V (IN. ³)
6	1	4.7	0.875	27.6
	3	1.7	2.38	28.2

Figure 6. Description of Injector Configuration 6.

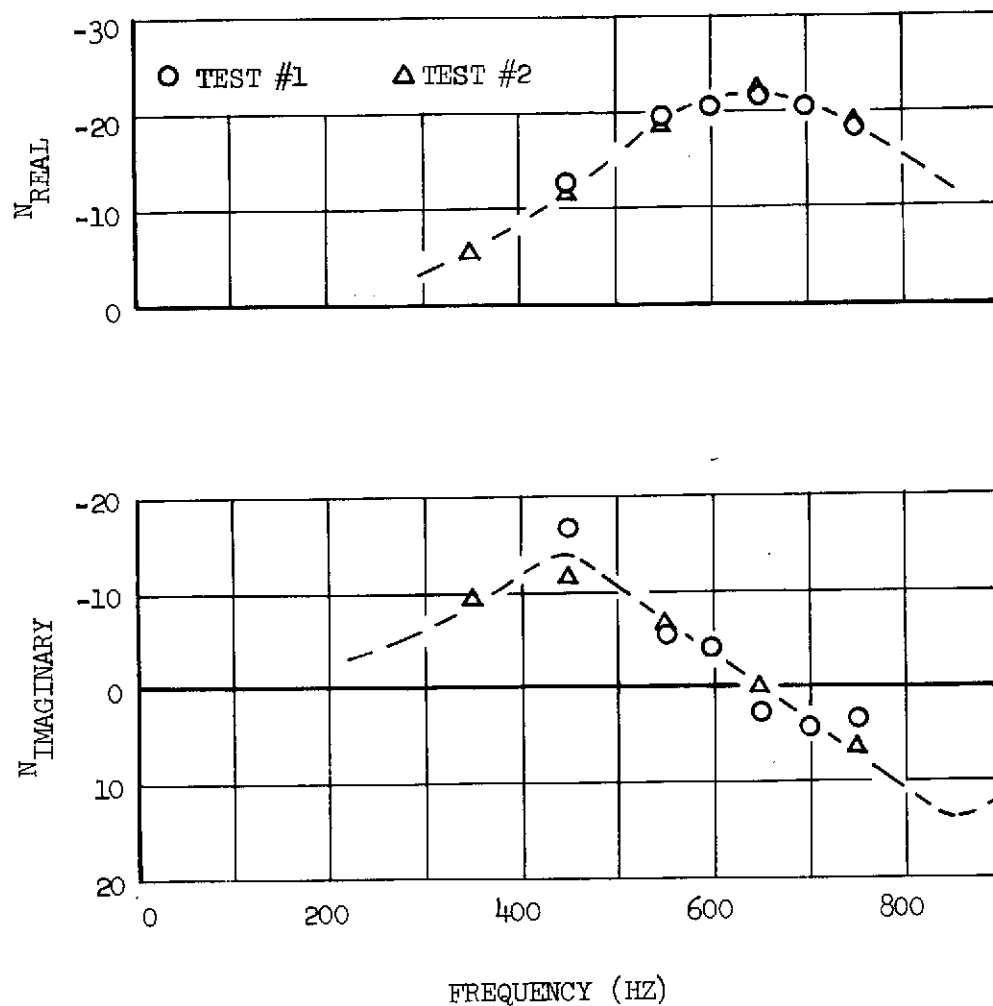


Figure 7. Repeatability of the Measured Response Factor Data.

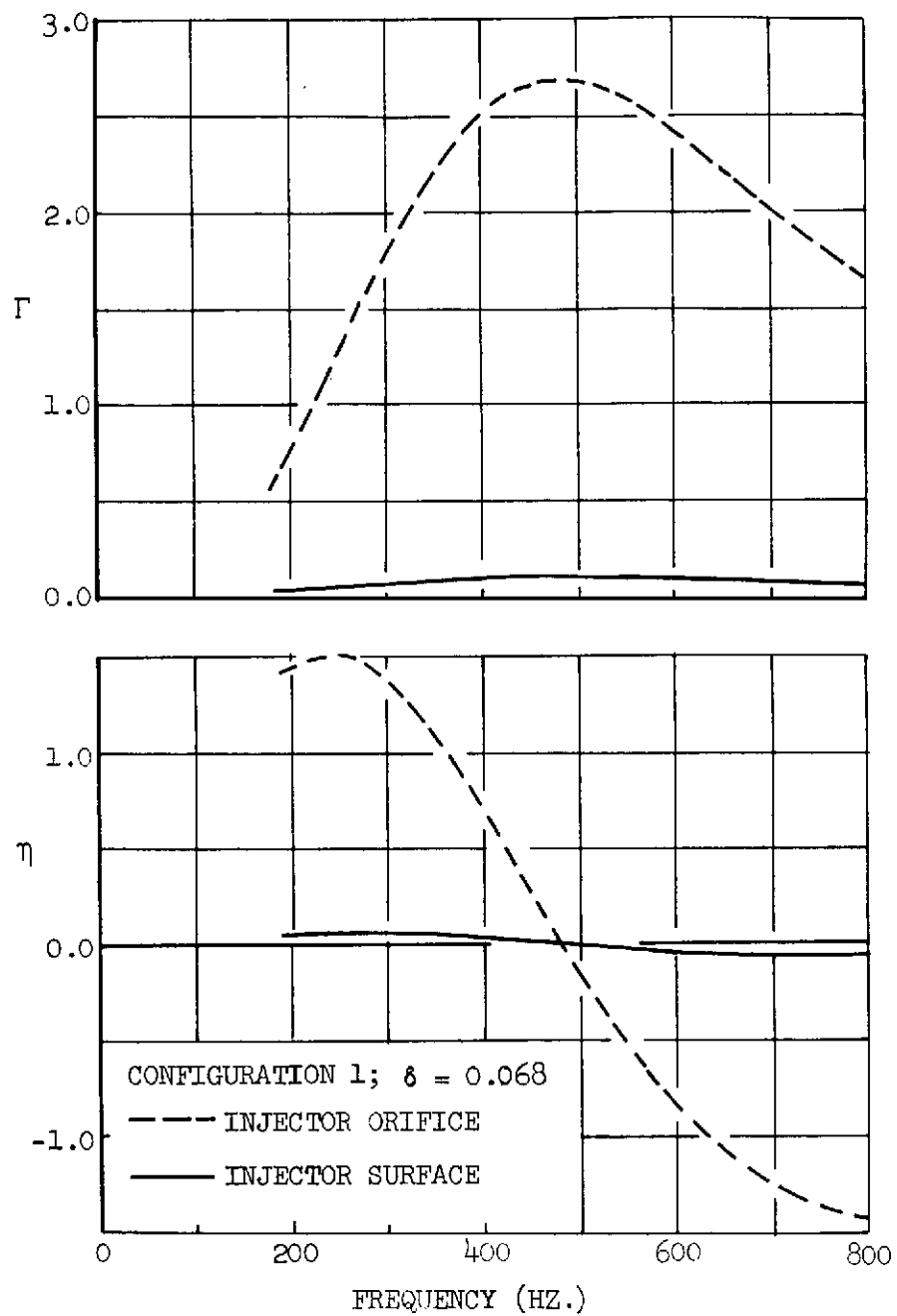


Figure 8. Predicted Admittances for the Injector Configuration 1.

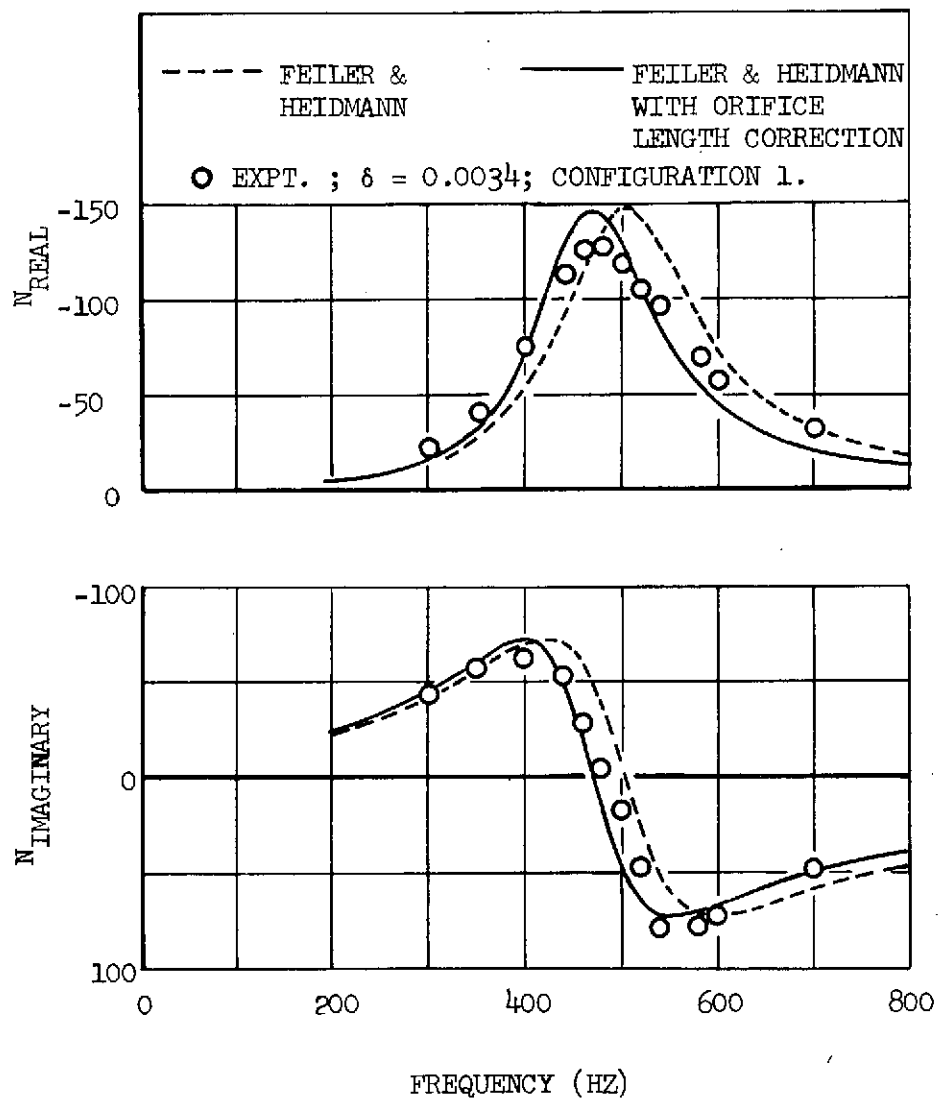


Figure 9. Feiler and Heidmann Predicted Response Factor Data with and without Orifice Length Correction.

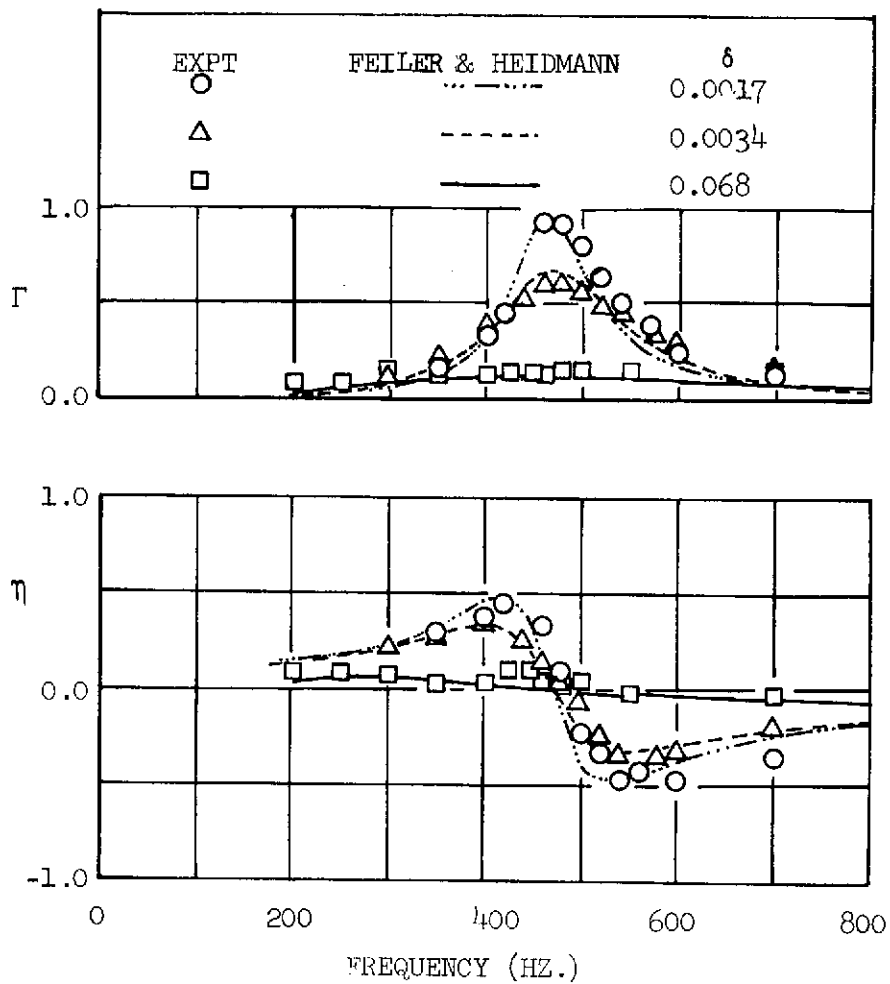


Figure 10. Frequency Dependence of the Surface Admittances of Injector Configuration 1.

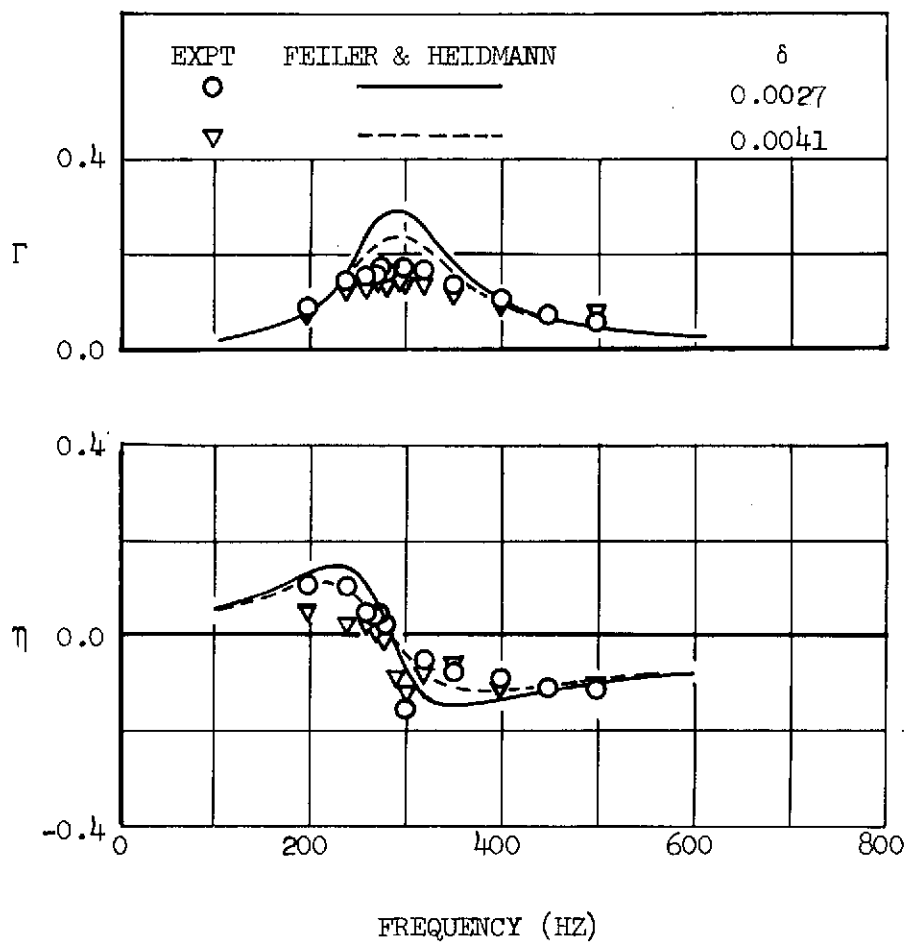


Figure 11. Frequency Dependence of the Surface Admittances of Injector Configuration 2.

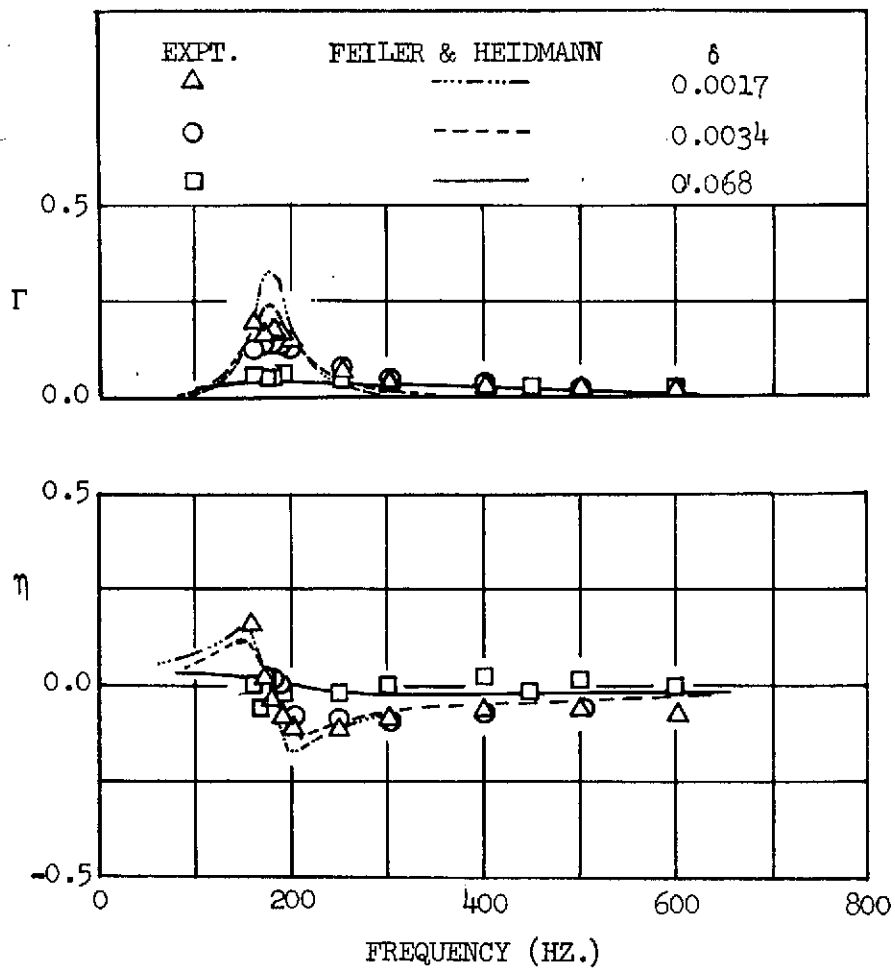


Figure 12. Frequency Dependence of the Surface Admittances of Injector Configuration 3.

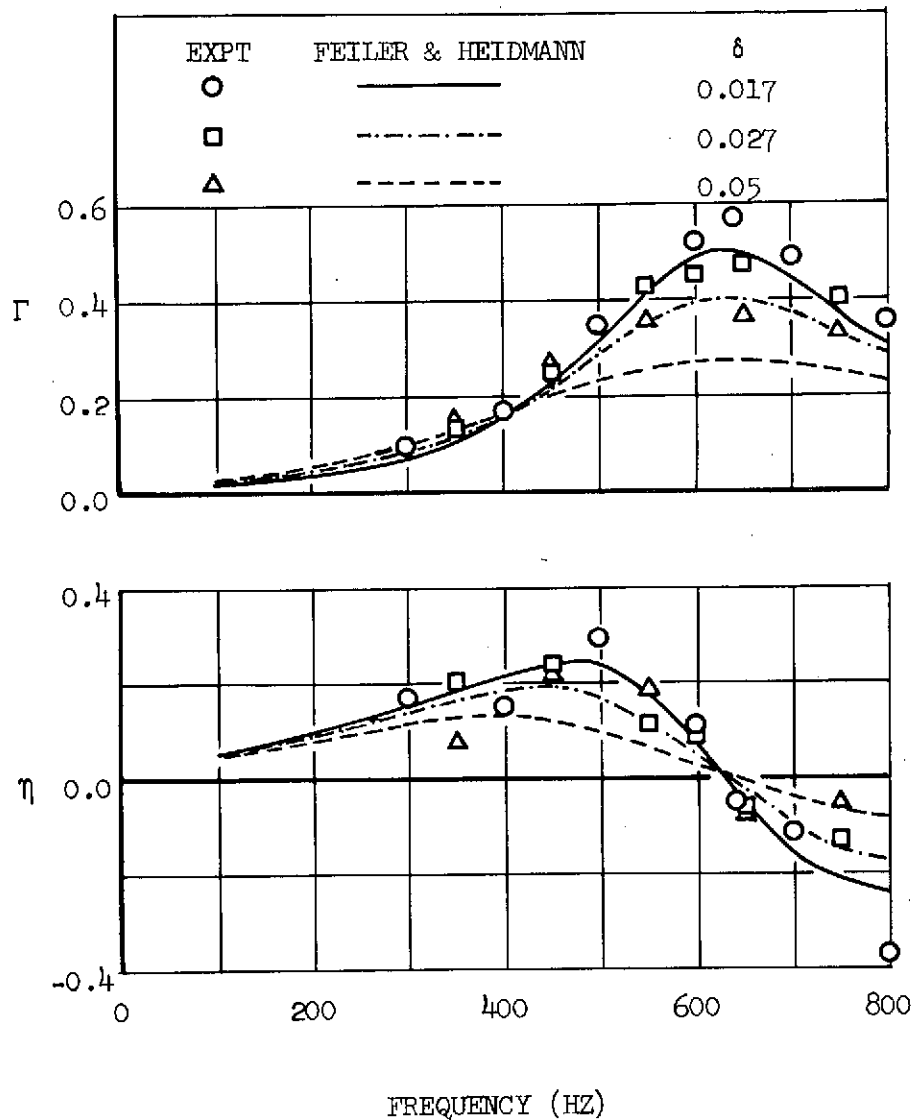


Figure 13. Frequency Dependence of the Surface Admittances of Injector Configuration 4.

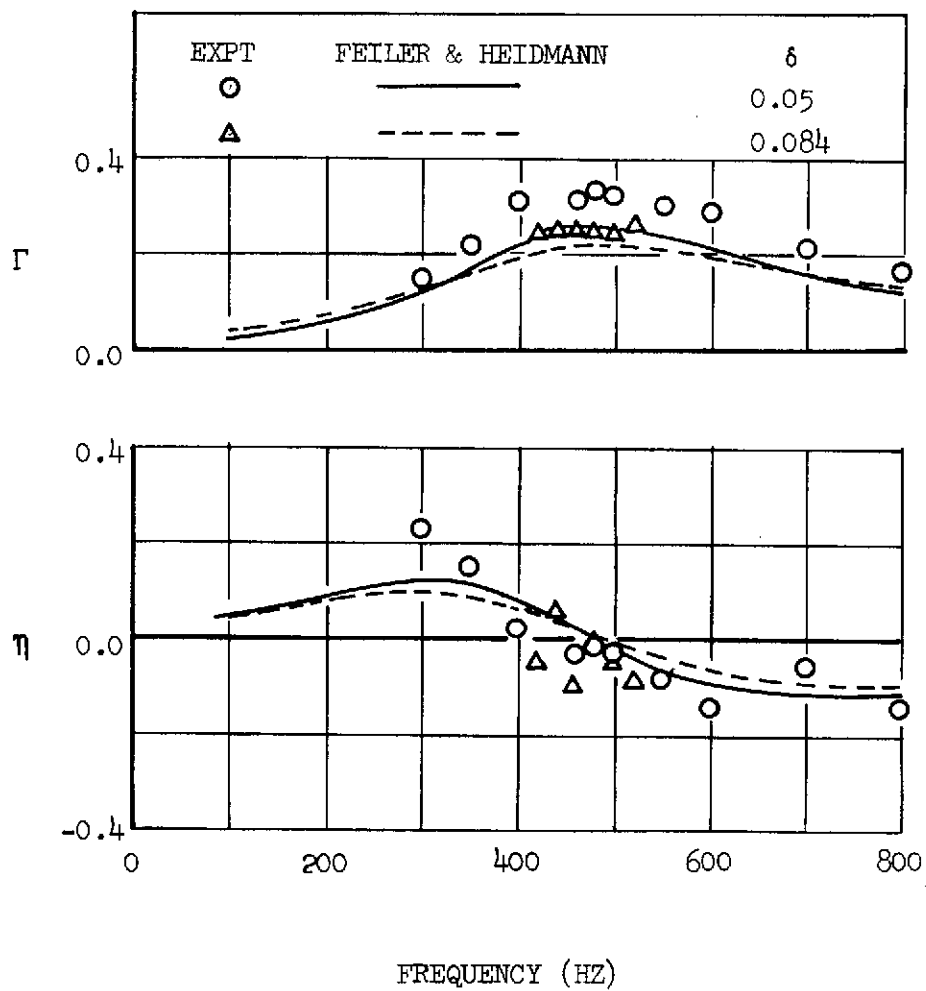


Figure 14. Frequency Dependence of the Surface Admittances of Injector Configuration 5.

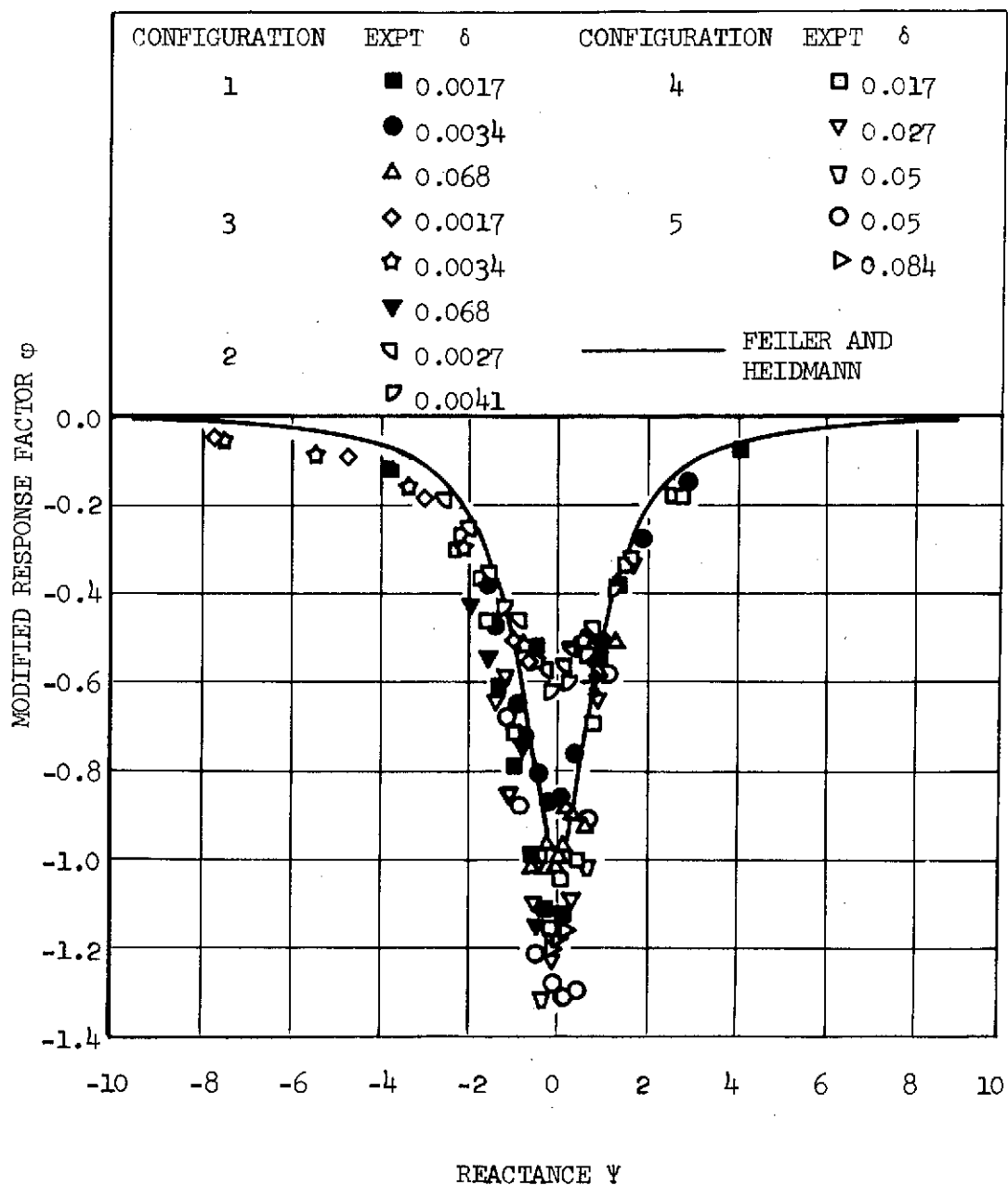


Figure 15. Generalized Response Factor Data Plotted Against Reactance.

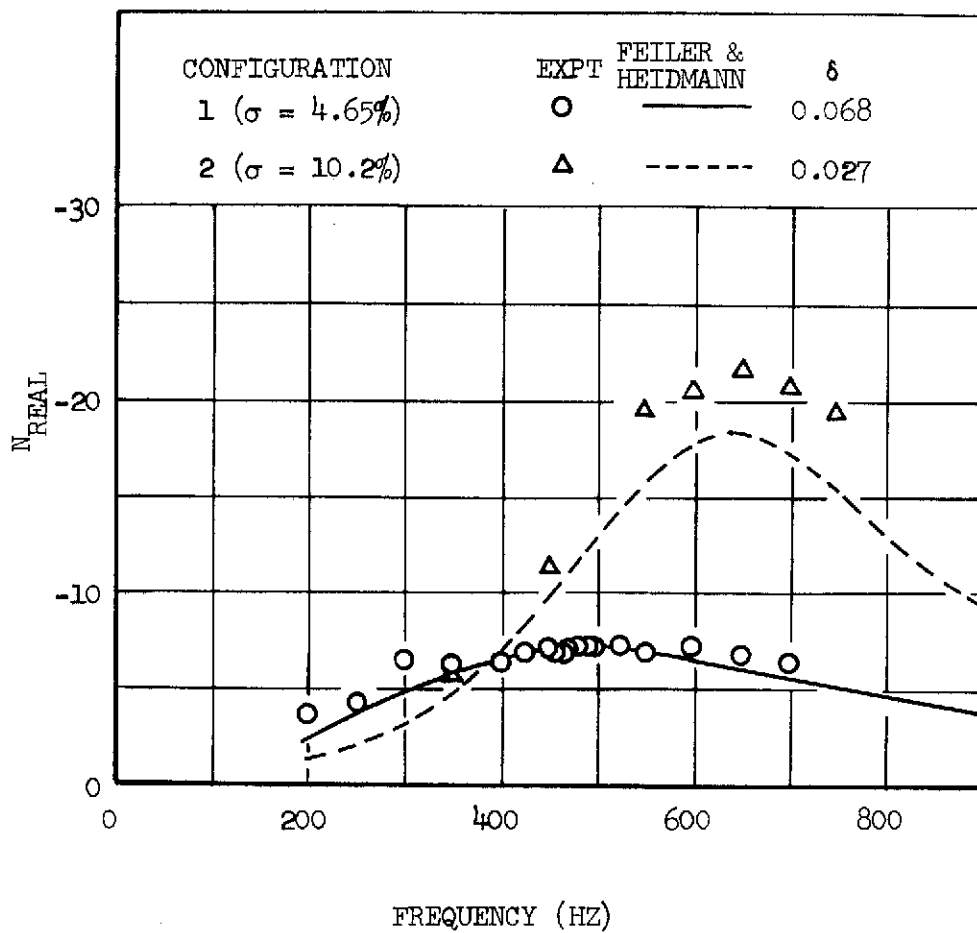


Figure 16. Effect of Open-Area Ratio on
Injector Response Factor.

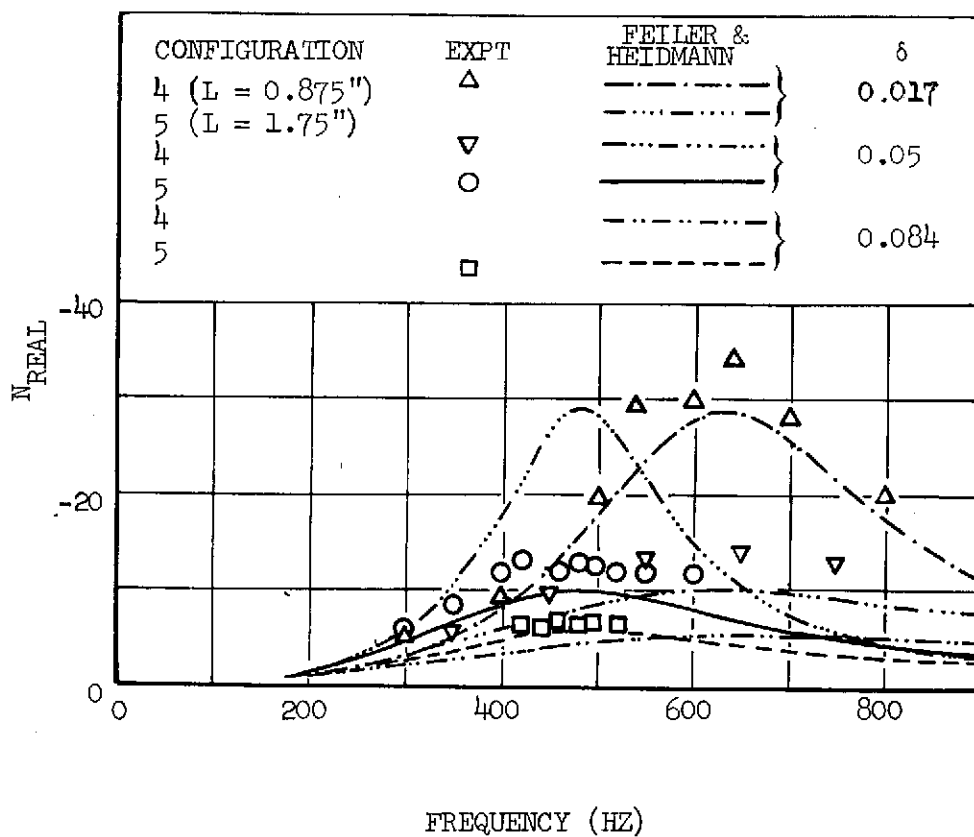


Figure 17. Effect of Orifice Length on
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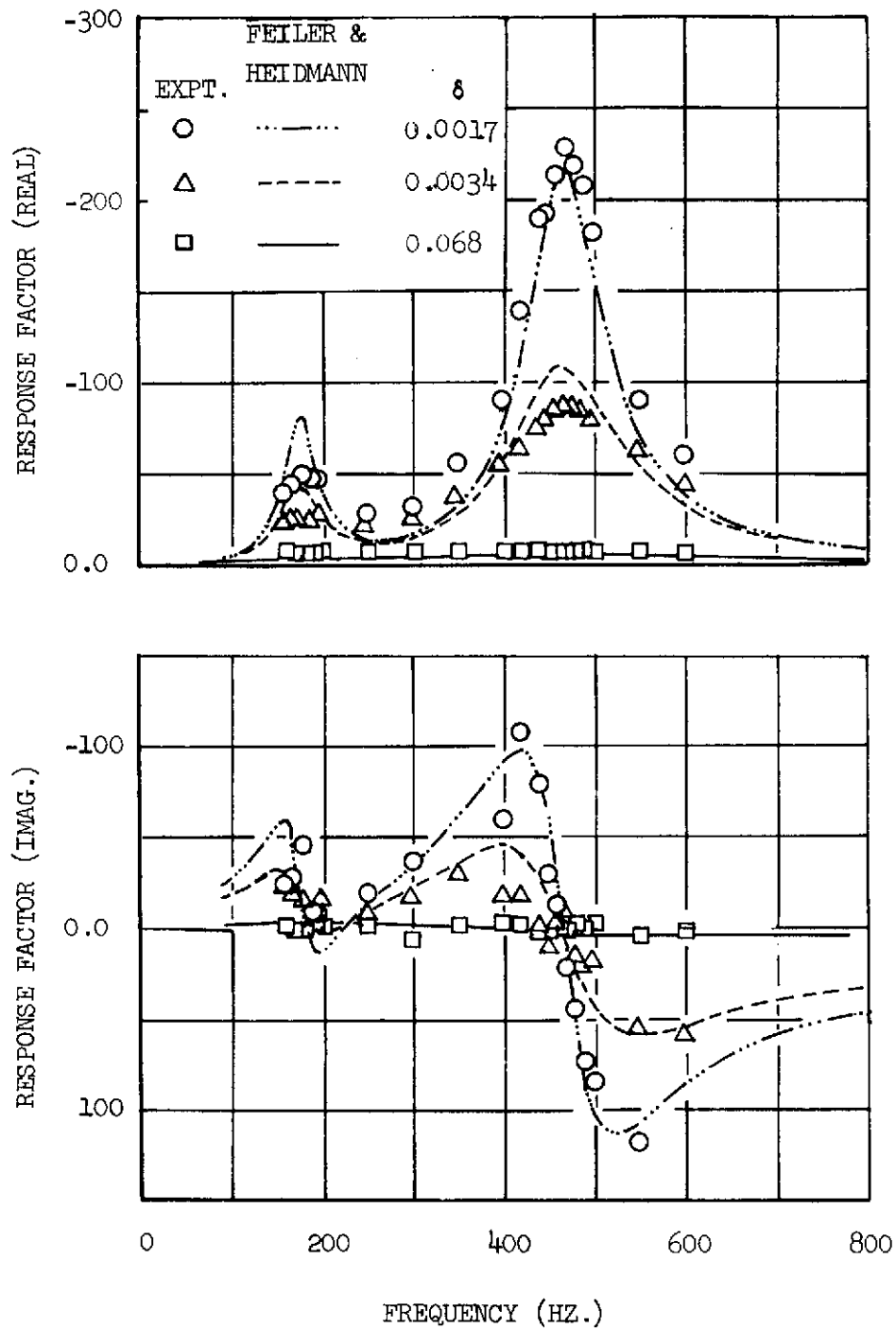


Figure 18. Frequency Dependence of Response Factors of Injector Configuration 6.

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